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Kalman Filtering in Navigation

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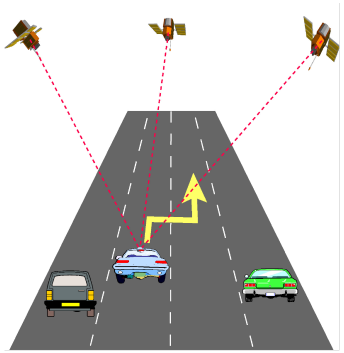
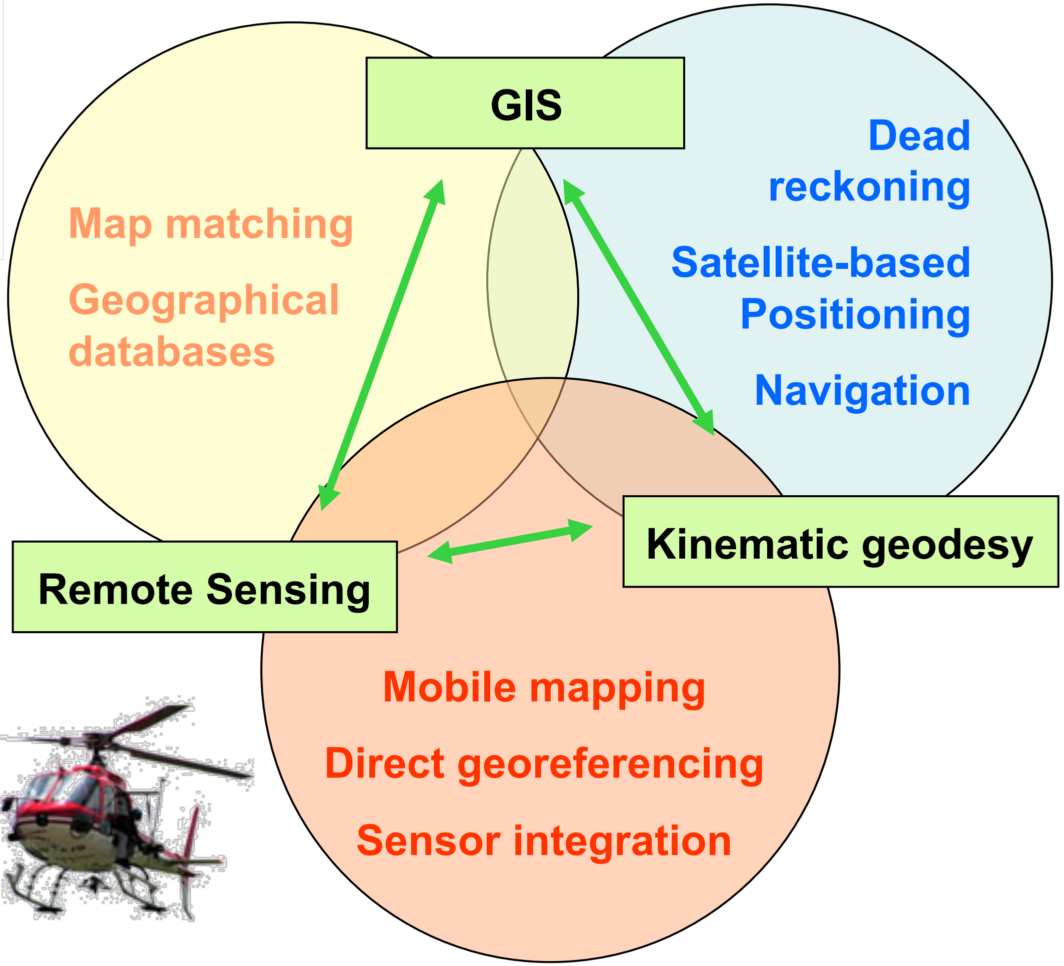
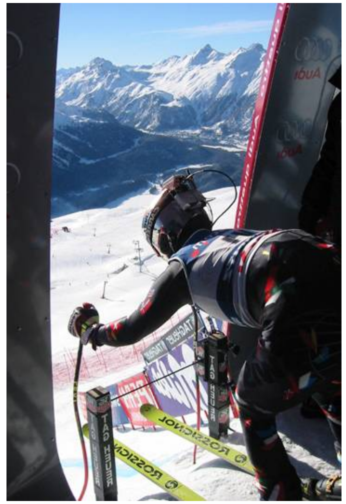
Presentation of the Geodetic Engineering Lab of EPFL



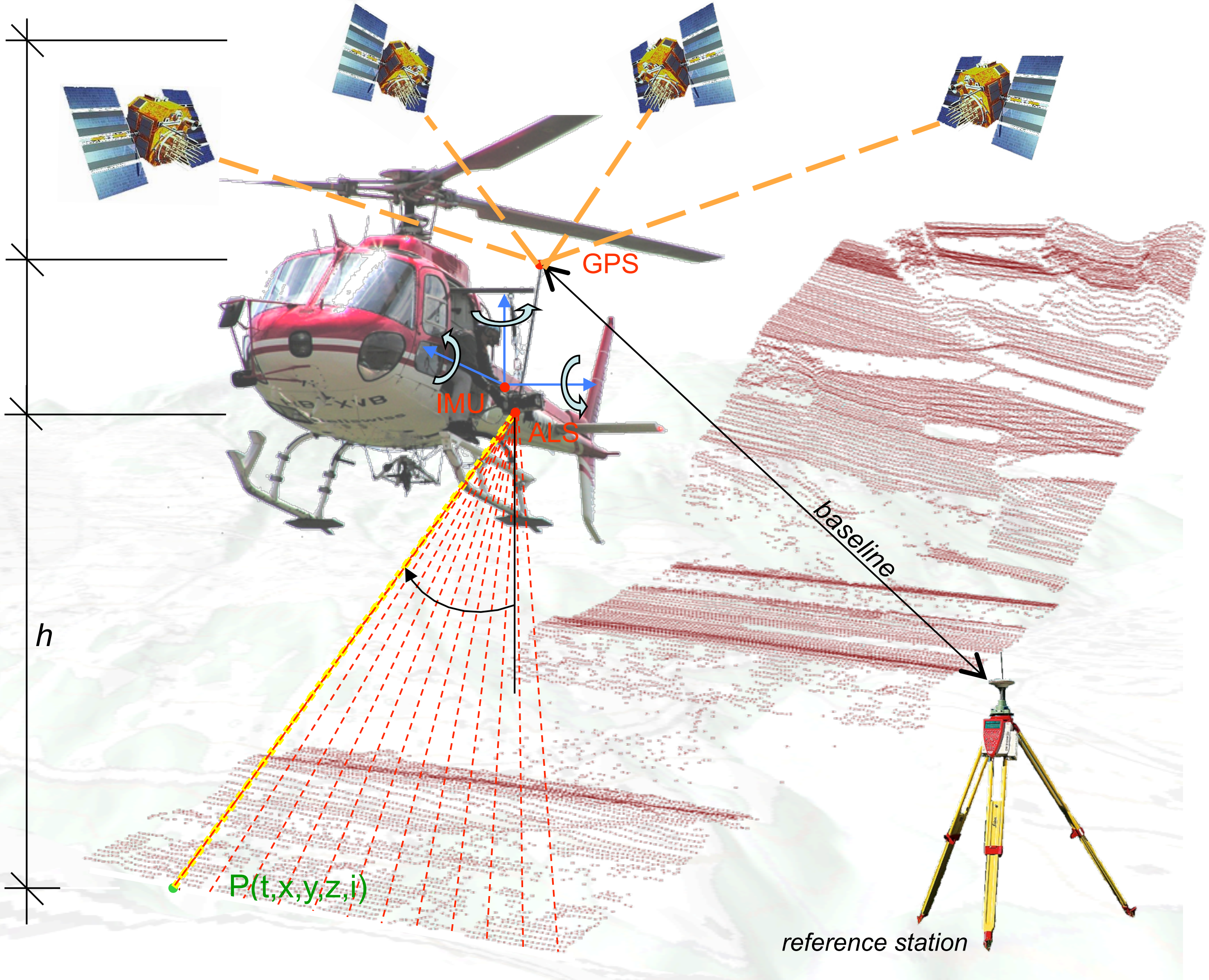


Photobus



Principles of Airborne Laser Scanning

GPS	Absolute position
IMU	Laser orientation
LiDAR	Range measure Encoder angle



Basics

The Kalman filter is a modelling and computation technique which is adequate to estimate parameters that change with time in a predictable manner, typically in navigation.

It may be regarded as an extension of the classical least squares, with:

- the possibility to combine various types of observations,
- a sequential structure,
- numerical advantages,
- aptitude for real time.

Rudolf E. Kalman, former professor of mathematics at ETH Zurich

Overview

- observation modelling
  - functional model
  - stochastic model
- movement modelling
  - functional model
  - stochastic model
- a few example
- interpretation of the results
- limitations
- alternatives

Least squares - parametric adjustment

$$\underset{nx1}{\check{\boldsymbol{\ell}}} = \mathbf{f}(\underset{nx1}{\check{\mathbf{x}}}) \qquad\qquad \boldsymbol{\ell} - \mathbf{v} = \mathbf{f}(\mathbf{x})$$

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathring{\mathbf{x}}) + \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\circ} \cdot (\mathbf{x} - \mathring{\mathbf{x}}) = \mathring{\boldsymbol{\ell}} + \mathbf{A} \cdot \delta\mathbf{x}$$

$$\underset{nx1}{\mathring{\mathbf{v}}} - \underset{nx1}{\mathbf{v}} = \underset{nxu}{\mathbf{A}}\,\underset{ux1}{\delta\mathbf{x}}$$

$$\underset{u\times 1}{\delta\hat{\mathbf{x}}} = \underset{u\times u}{\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}} \cdot \underset{u\times 1}{\mathbf{A}^T\mathbf{P}\,\mathring{\mathbf{v}}}$$

$$\hat{\mathbf{x}} = \mathring{\mathbf{x}} + \delta\hat{\mathbf{x}} \qquad\qquad \mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}$$

Sequential Least Squares - Bayes

We condense the knowledge gained via the previous observations in a set of parameters with their covariance matrix. Then we add a new set of observations.

$$\begin{bmatrix} \delta\tilde{\mathbf{x}} \\ \overset{\circ}{\mathbf{v}} \end{bmatrix} - \begin{bmatrix} \mathbf{v}_{\mathbf{x}} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathrm{I} \\ \mathbf{A} \end{bmatrix} \cdot \delta\mathbf{x} \qquad \begin{bmatrix} \mathbf{P}_{\tilde{\mathbf{x}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{bmatrix}$$

$$\hat{\mathbf{x}} = \tilde{\mathbf{x}} + \left(\mathbf{P}_{\tilde{\mathbf{x}}} + \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1} \cdot \mathbf{A}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{v}}$$

$$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \left(\mathbf{P}_{\tilde{\mathbf{x}}} + \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}$$

De Bayes à Kalman

- L'identité suivante est classique en algèbre linéaire.

$$\left(\mathbf{B}+\mathbf{C}^{\mathrm{T}}\mathbf{D}\mathbf{C}\right)^{-1}=\mathbf{B}^{-1}-\mathbf{B}^{-1}\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{B}^{-1}\mathbf{C}^{\mathrm{T}}+\mathbf{D}^{-1}\right)^{-1}\mathbf{C}\mathbf{B}^{-1}$$

- Pour les termes de la forme de Bayes, on obtient:

$$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}=\left(\mathbf{P}_{\tilde{\mathbf{x}}}+\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}=\mathbf{P}_{\tilde{\mathbf{x}}}^{-1}-\mathbf{P}_{\tilde{\mathbf{x}}}^{-1}\mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{P}_{\tilde{\mathbf{x}}}^{-1}\mathbf{A}^{\mathrm{T}}+\mathbf{P}^{-1}\right)^{-1}\mathbf{A}\mathbf{P}_{\tilde{\mathbf{x}}}^{-1}$$

- On pose: $\mathbf{K}=\mathbf{Q}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{Q}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}+\mathbf{Q}_{\ell\ell}\right)^{-1}$

- et l'on obtient: $\hat{\mathbf{x}}=\tilde{\mathbf{x}}+\mathbf{K}\cdot\left(\overset{\circ}{\mathbf{v}}-\mathbf{A}\delta\tilde{\mathbf{x}}\right)$

- ainsi que: $\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}=\left(\mathrm{I}-\mathbf{K}\mathbf{A}\right)\cdot\mathbf{Q}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}\cdot\left(\mathrm{I}-\mathbf{K}\mathbf{A}\right)^{\mathrm{T}}+\mathbf{K}\mathbf{Q}_{\ell\ell}\mathbf{K}^{\mathrm{T}}$

Modelling the movement

The parameters change between epochs. Using the adjusted values at epoch t_0, we predict their values at epoch t.

$$\tilde{\mathbf{x}} = \mathbf{\Phi} \cdot \hat{\mathbf{x}}_0$$

The uncertainty in the parameters at is propagated and noise is added.

$$\mathbf{Q}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = \mathbf{\Phi} \cdot \mathbf{Q}_{\hat{\mathbf{x}}_0\hat{\mathbf{x}}_0} \cdot \mathbf{\Phi}^{\mathrm{T}} + \mathbf{Q}_{\mathbf{ww}}$$

Filtrage 1-D

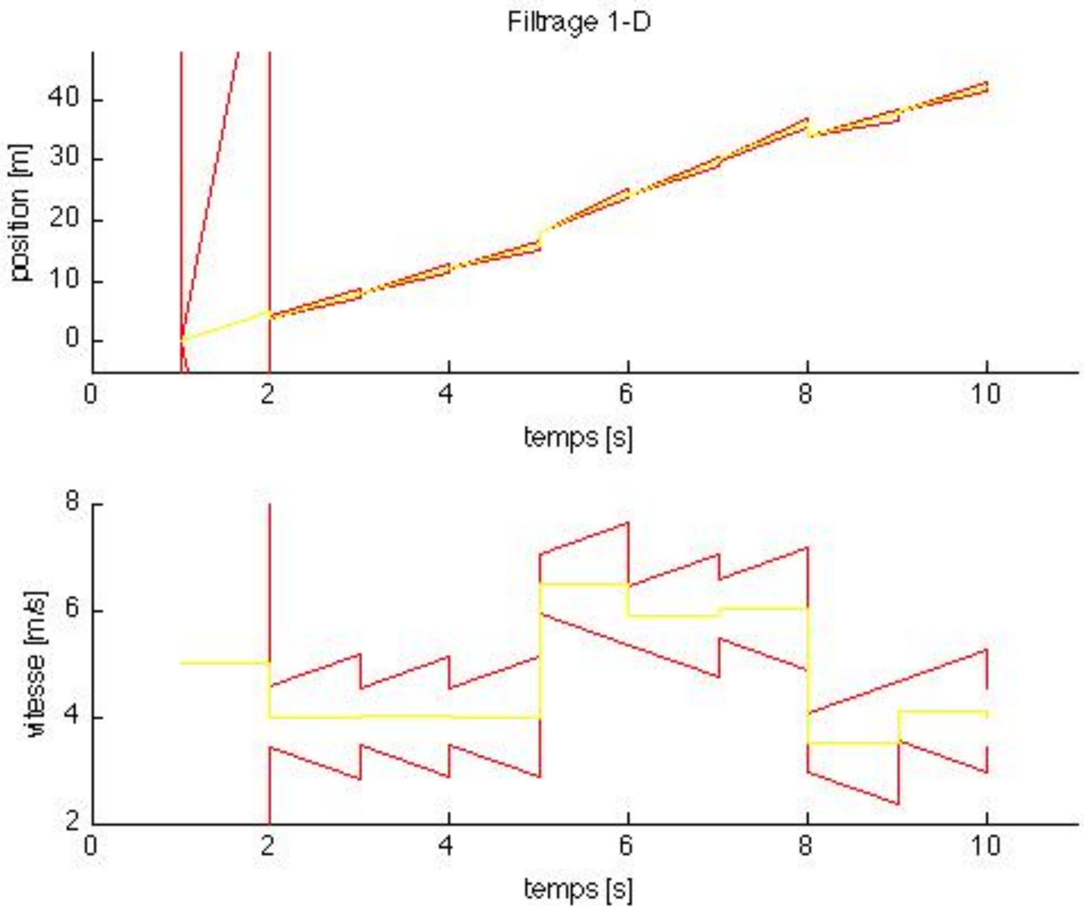
modèle des observations

$$\ell_x - v = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ v \end{bmatrix} \qquad \mathbf{Q}_{\ell\ell} = q_{\ell\ell} = \sigma_x^2$$

modèle du mouvement

$$\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ v_0 \end{bmatrix} \qquad \mathbf{Q}_{ww} = \begin{bmatrix} q_v \Delta t^3/3 & q_v \Delta t^2/2 \\ q_v \Delta t^2/2 & q_v \Delta t \end{bmatrix}$$

Filtrage 1-D - résultats



Compte-tours avec baromètre

Modèle des observations

A chaque tour de roue, on mesure le temps et la pression.

paramètres: D = distance parcourue
v = vitesse
H = altitude
m = pente

$$\begin{cases} \ell_{\Delta t} - v_{\Delta t} = c/v \\ \ell_p - v_p = f(H) \end{cases}$$

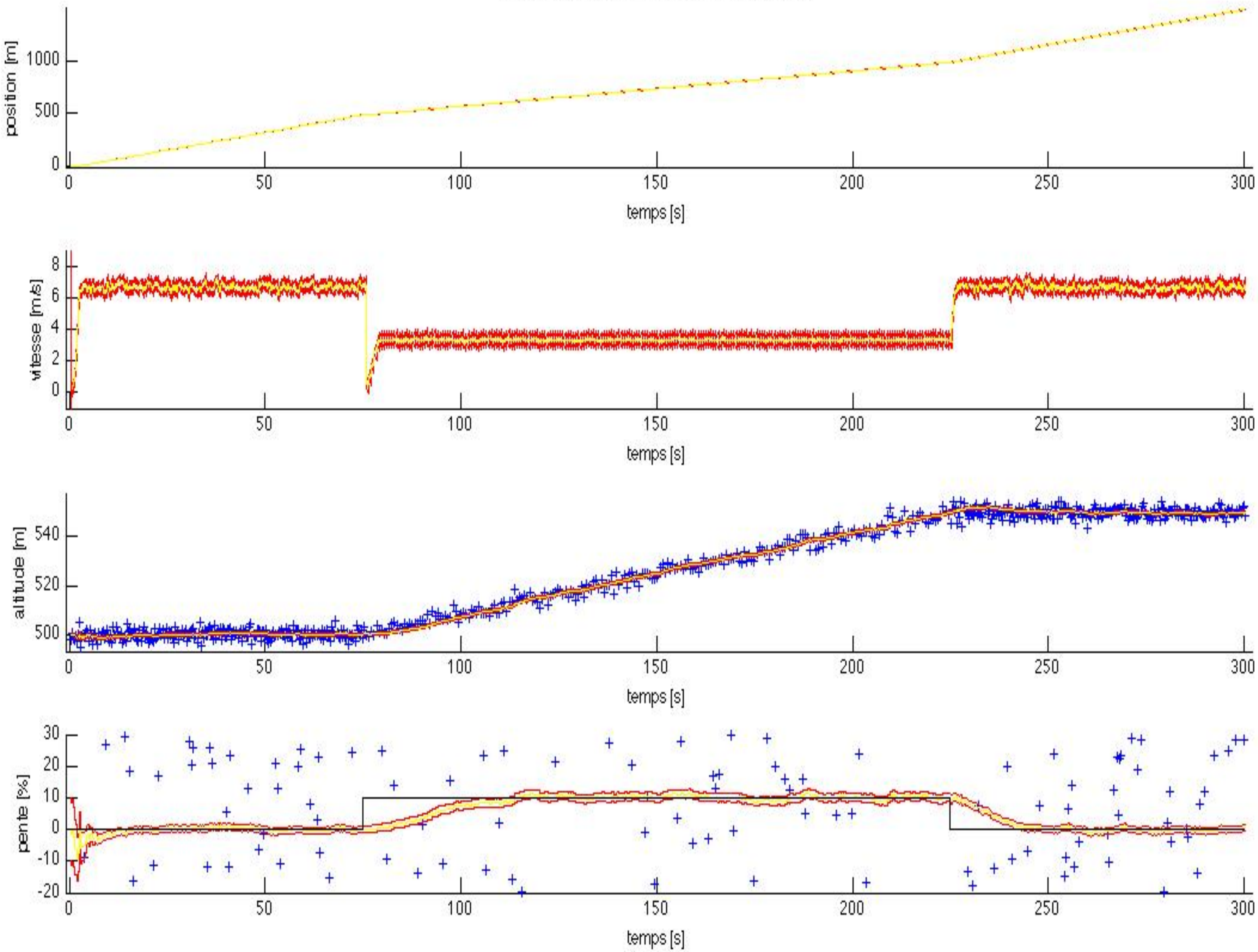
$$\mathbf{Q}_{\ell\ell} = \begin{bmatrix} \sigma^2_{\Delta t} & 0 \\ 0 & \sigma^2_p \end{bmatrix}$$

Modèle du mouvement extension du filtrage 1-D

$$\begin{bmatrix} D \\ v \\ H \\ m \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & c/100 \\ 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} D_0 \\ v_0 \\ H_0 \\ m_0 \end{bmatrix}$$

$$\mathbf{Q_{ww}} = \left[\begin{array}{cc|cc} q_v \Delta t^3/3 & q_v \Delta t^2/2 & 0 & 0 \\ q_v \Delta t^2/2 & q_v \Delta t & 0 & 0 \\ \hline 0 & 0 & (c/100)^2 \cdot (q_m \Delta t^3/3) & (c/100) \cdot (q_m \Delta t^2/2) \\ 0 & 0 & (c/100) \cdot (q_m \Delta t^2/2) & q_m \Delta t \end{array}\right]$$

Compte-tours avec baromètre - résultats



Limitations and alternatives

- Like all least squares algorithms, the Kalman filter is sensitive to large residuals, that is, it does not work well with many outliers.
- If the movement model is loose, the filter reacts more quickly to a change in the trajectory, but the latter will look unstable.
- If the movement model is stiff, the trajectory will look smooth, but the filter reacts slowly to a change.
- The models can be adapted according to the noise level in the results, but after a smooth period, it will take time to detect a sudden change.
- We look forward to the presentation of the particle filters!

Conclusion

- KF is a very flexible approach for sequential estimation, thanks to the clear separation into observation and movement models.
- It works fine with good measurements and a well behaved movement.
- For measurements with many outliers, or for unpredictable movements, alternatives are advisable.